**Chapter 6**

**Power Series**

**6.2 Properties of Power Series**

**Section Exercises**

63. If  and , find the power series of  and of 

Answer:  and .

**In the following exercises, use partial fractions to find the power series of each function.**

65. 

Answer: 

67. 

Answer: 

**In the following exercises, express each series as a rational function.**

69. 

Answer: 

71. 

Answer: 

**The following exercises explore applications of annuities.**

73. Calculate the present values *P* of an annuity in which $10,000 is to be paid out annually for a period of 20 years, assuming interest rates of , , and 

Answer: where . Then When ,  When ,  When , 

75. Calculate the annual payouts *C* to be given for 20 years on annuities having present value $100,000 assuming respective interest rates of , , and 

Answer: In general,  for *N* years of payouts, or  For  and , one has  when ;  when ; and when 

77. Suppose that an annuity has a present value . What interest rate *r* would allow for perpetual annual payouts of $50,000?

Answer: In general, . Thus, 

**In the following exercises, express the sum of each power series in terms of geometric series, and then express the sum as a rational function.**

79.  (*Hint:* Group powers *x*3*k*, , and )

Answer: 

81.  (*Hint:* Group powers *x*3*k*, , and )

Answer: 

**In the following exercises, find the power series of  given *f* and *g* as defined.**

83. , 

Answer: , so  and 

85. 

Answer:  so and 

**In the following exercises, differentiate the given series expansion of *f* term-by-term to obtain the corresponding series expansion for the derivative of *f*.**

87. 

Answer: The derivative of  is .

**In the following exercises, integrate the given series expansion of  term-by-term from zero to *x* to obtain the corresponding series expansion for the indefinite integral of **

89. 

Answer: The indefinite integral of is 

**In the following exercises, evaluate each infinite series by identifying it as the value of a derivative or integral of geometric series.**

91. Evaluate  as  where 

Answer: ; so 

93. Evaluate  as  where 

Answer: ; so .

**In the following exercises, given that , use term-by-term differentiation or integration to find power series for each function centered at the given point.**

95.  centered at  (*Hint:*)

Answer: 

97.  at 

Answer: 

99.  at 

Answer: 

101.  where 

Answer: Term-by-term integration gives 

103. **[T]** Subtract the infinite series of  from  to get a power series for  Evaluate at  What is the smallest *N* such that the *N*th partial sum of this series approximates  with an error less than 0.001?

Answer: We have so . Thus,  When  we obtain  We have , while  and ; therefore, 

**In the following exercises, using a substitution if indicated, express each series in terms of elementary functions and find the radius of convergence of the sum.**

105. 

Answer:  so The radius of convergence is equal to 1 by the ratio test.

107. using

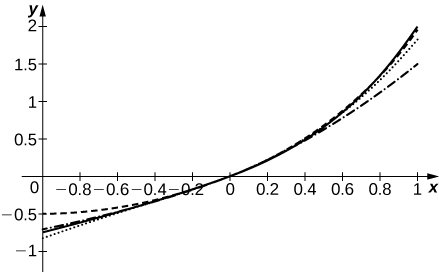
Answer: If , then . If , then  when . So the series converges for all .

109. Differentiate the series  term-by-term to show that  is equal to its derivative.

Answer: Answers will vary.

111. **[T]** Suppose that the coefficients *an* of the series  are defined by the recurrence relation . For and , compute and plot the sums  for  on 

Answer:



The solid curve is *S*5. The dashed curve is *S*2, dotted is *S*3, and dash-dotted is *S*4

113. **[T]** Given the power series expansion , determine how many terms *N* of the sum evaluated at  are needed to approximate  accurate to within 1/1000. Evaluate the corresponding partial sum .

Answer: When  ,. Since , one has  whereas ; therefore, .

115. **[T]** Recall that . Assuming an exact value of , estimate  by evaluating partial sums  of the power series expansion  at . What is the smallest number *N* such that  approximates *π* accurately to within 0.001? How many terms are needed for accuracy to within 0.00001?

Answer: . One has  and  so  is the smallest partial sum with accuracy to within 0.001. Also,  while  so  is the smallest *N* to give accuracy to within 0.00001.

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